Effective cloning phenomenon in T-shaped Josephson transmission lines

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Abstract

The propagation of single flux quanta in T-shaped Josephson junctions gives rise to the flux cloning phenomenon. We have studied numerically the dynamics of flux cloning in cases of extended Josephson junctions. The changing thicknesses of T-junctions leads to new and interesting effects in terms of their dynamics. We have found out that when an additional Josephson transmission line is larger than the main Josephson transmission line, numerical simulations do not show the cloning phenomenon and soliton is reflected when it approaches the T junction. We have investigated conditions at which flux cloning occurs when both widths, W and W_0, are changing.

Key Words: Josephson junction, Single vortex, T-junctions, flux cloning, widths.

1.Introduction:

Vortices occur naturally in a wide range, from macroscopic to microscopic scales. They play an important role in different branches of physics. In superconductivity, vortices can be divided into two types. The first vortex, the Abrikosov vortex, appears in certain type II superconductors. The second vortex, the Josephson vortex, which is often referred to as fluxon or soliton, exists in a Josephson junction. Although Josephson vortices are different from Abrikosov vortices, they exhibit similar behavior (Gulevich and Kusmartsev, 2007a).

The dynamics of Josephson junctions and solitons arising in Josephson transmission line (JTL), are known in detail (Barone and Paterno, 1982). Recently, there has been a great deal of interest in the study of single flux quanta on the JTLs, because of their potential applications in superconducting electronics and digital logic circuitry. Josephson junctions, which are made up of two superconductors separated by a thin insulating layer, are being investigated as potential switching elements for ultra-fast computers because they switch very rapidly, at extremely low power levels.

Strictly speaking, the standard description of fluxon dynamics in JTL is based on the (2+1)-dimensional unperturbed sine-Gordon equation. With its boundary conditions (Barone and Paterno, 1982), this description is extremely accurate:

$$\phi_{xx} + \phi_{yy} - \phi_{tt} - \sin\phi = 0 \qquad (1)$$

where $\varphi(x, y, t)$ is the superconducting phase difference, and the subscripts denote partial derivatives of x, y *and* time t. The exact soliton solutions of the equation have been given as

$$\varphi(x,y,t) = 4\tan^{-1}\left(\exp\frac{x - x_0 - ut}{\sqrt{1-u^2}}\right) \qquad (2)$$

With the energy

$$E = \int_{-\infty}^{\infty} dx \int_{0}^{y} dy \left[\frac{\varphi_t^2}{2} + \frac{\varphi_x^2}{2} + \frac{\varphi_y^2}{2} + 1 - \cos\varphi \right] = \frac{8W}{\sqrt{1-u^2}} \tag{3}$$

where u is a velocity of soliton propagating in a straight two-dimensional strip of width W, which is located at $x = x_0$ for $t = 0$. The velocity u (normalized to the Swihart velocity $\bar{c}$, the maximum electromagnetic propagation velocity in the junction) may assume values $-1 \le u < 1$. At $u = 0$, the rest energy equals 8 W, which is identified with the normalized rest mass of the soliton.

2. Flux cloning

T-shaped junctions of JTL are a type of multiple JTL connected together. These are of great potential use on logic electronic circuits. Basically, T junctions are divided into two different perpendicular widths, a main JTL of width W_0 along the $x-$axis, and an additional JTL of width W along the $y-$axis. They are connected to form a T junction (See Fig. 1). Flux cloning is a newly discovered phenomenon that occurs as a result of T-shaped junctions (Gulevich and Kusmartsev, 2006; 2007b). Flux cloning has been used in special devices to create fluxons without the application of an external magnetic field and may be used in generator of T-rays (Gulevich et al, 2008). T-ray is a new and invisible electromagnetic radiation of a safe, non-ionizing kind.

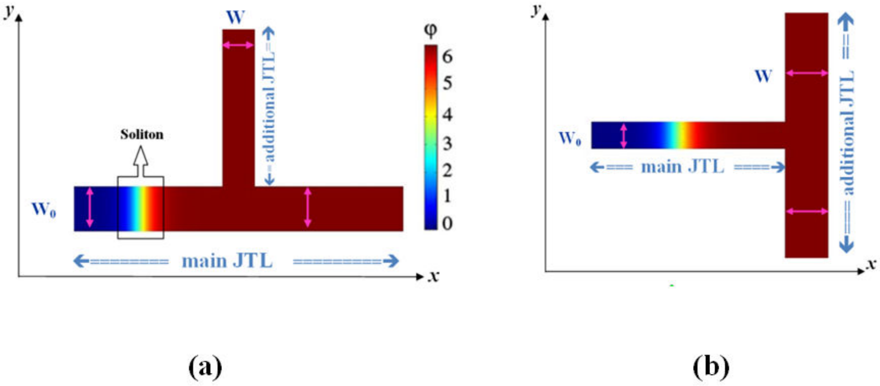


Figure (1): (a) T-junctions, (b) rotated T-junctions, in Josephson transmission lines that represent the numerical simulation of soliton. The color scale represents the superconducting phase difference φ..

Flux cloning means that under certain conditions a propagating single Josephson vortex can create a new Josephson vortex. This phenomenon has been explained by Gulevich and Kusmartsev (2006) who studied the dynamic behavior of solitons in T junctions, as shown in Fig.1 (a), and pointed out that in the connecting area, dependent on the velocity of the soliton, two different types of behaviour are possible - either reflection from the T junction (Fig. 2(a)) or flux cloning (Fig. 2(b)). When the mother vortex in the MJTL approaches the fork and satisfies certain conditions, it splits to generate a new baby vortex in the AJTL. In addition, the mother vortex will still propagate continuously in the main JTL. There are two conditions that need to exist to satisfy the flux cloning dependence on the velocity of the soliton and the bias current. The first condition, the critical velocity is

$$u_c = \frac{\sqrt{W(W+2W_0)}}{W+W_0} \tag{4}$$

The second condition, the critical driving current is

$$\gamma_c = \frac{4W}{\pi(2W_0+W)} \tag{5}$$

Otherwise, the soliton will be reflected if u and γ are less than u_c and γ_c, respectively.

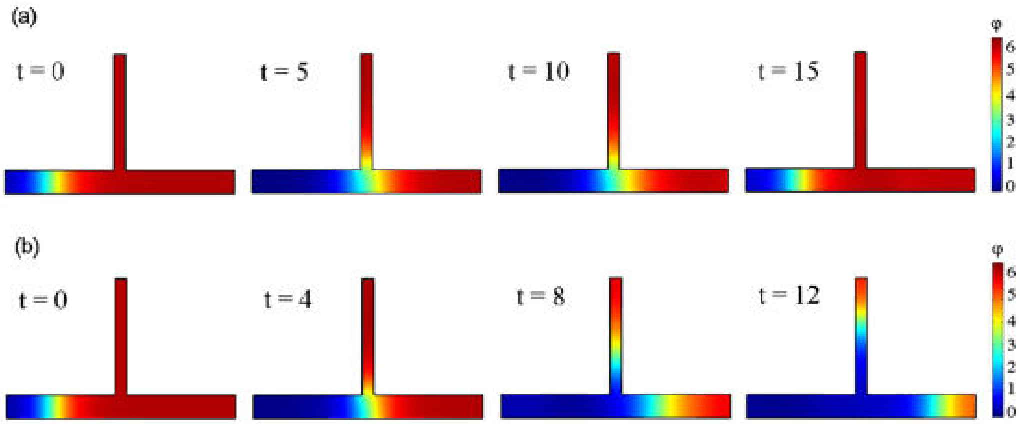


Figure (2): Numerical simulations of the superconducting phase difference (a) Reflection of an incident fluxon propagating without cloning and (b) cloning of a fluxon propagating.

Eq. (4) leads to the situation that if the value of the critical velocity (u_c) satisfies the condition $0 \leq u < 1$, the flux cloning occurs and does not depend on the value of the widths. Therefore, W and W_0 do not have any effect. The purpose of the present work is to undertake a numerical study of the variety of dynamic behaviours of flux cloning that can be expected in extended Josephson Junctions system. In addition, the relation between the thicknesses, W and W_0, will be investigated to satisfy flux cloning. In our numerical investigations, assuming both W_0 and $W \leq 1$ in units of Josephson penetration depth (λ_J), we have studied the effect of widths W and W_0 on the flux cloning in the case of an absence of the driving current. In order to understand the behaviour of the Josephson vortex in extended Josephson Junctions we have limited our analysis to the change in the width of the additional JTL only, and have focused on the behaviour in three cases $W > W_0$, $W = W_0$ and $W < W_0$.

3. Numerical studies of flux cloning:

Our numerical simulations of the flux cloning dynamics were undertaken by using the COMSOL program, which the finite element numerical technique package FEMLAB utilises. The time-dependent sine-Gordon-equation with boundaries were included as one of its standard equation-based models. The dynamics in Josephson junctions is demonstrated by changing the value of the superconducting phase differences which are represented in the program by a change of colour, with the dark blue colour representing the minimum value of the phase ϕ and red representing the maximum value of ϕ. In the soliton case, the behaviour can be noted numerically by changing the value of ϕ, with time as the minimum ϕ equal to 0, the maximum ϕ equal to 2π and the intermediate colour between these values representing the vortices (see the colour scale in Fig. 1). The boundary conditions are $\phi_x = 0$ at both ends, $x = 0$ and $x = L$ at the junction. In all our numerical simulations, we have used initial conditions with the following forms: for the phase $\varphi(x, y, t)\big|_{t=0} = \varphi_{soliton}(x)$ and $\partial\varphi(x, y, t)/\partial t\big|_{t=0} = -u\,\partial\varphi_{soliton}(x)/\partial x$ for its time derivative, where $\varphi_{soliton}(x) = \varphi(x, y, 0)$. The initial soliton position was $x_0 = -3$.

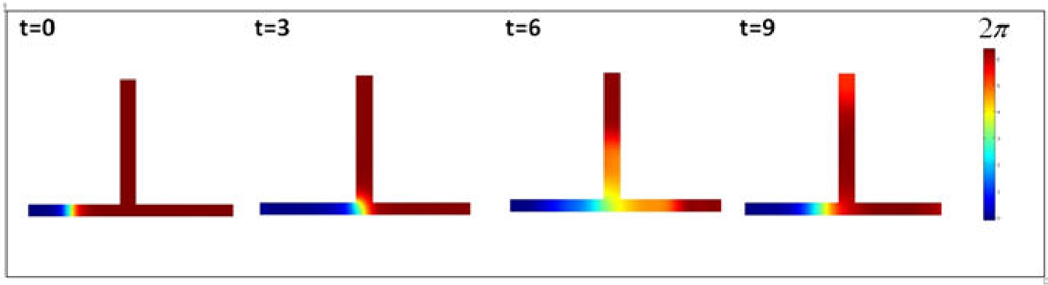
The numerical simulation of the behavior of the flux was studied when $W_0 = 1$ and $W = 0.5$. The analytical and numerical results of u_c and W/W_0 have coincided strikingly (Gulevich and Kusmartsev, 2006 and 2007), as shown in Fig. 3. In our study, assuming $W_0 = 0.6$ and $W \leq 1$, when we changed the width to $W > W_0$, the critical velocity is calculated to satisfy the flux cloning. However, the numerical simulations illustrate that here a mother vortex struggles to create a baby vortex. Under these circumstances, the cloning phenomenon is not occurring. Although the vortex is moving fast and it has a huge energy, it still does not give birth to a new vortex. Consequently, the mother vortex is reflected when it approaches the T junction (see Fig.3 (a)). Similarly, the same strange result is obtained when $W = W_0$ (see Fig.3 (b)). These results may be due to another kind of excitation entitled breather soliton. Meanwhile, when $W < W_0$, the mother vortex can give birth to a new vortex and flux cloning occurs as shown in Fig.4.

4. Conclusions:

In conclusion, our simulations show that flux cloning will occur if it moves to narrow AJTLs. In other words, the ratio between the widths, W and W_0, should be less than one. Therefore, the new condition should be added to the flux cloning phenomena. This third condition relates to the width of the MJTL and AJTLs as

$$W < W_0 \quad \text{or} \quad \frac{W}{W_0} < 1 \tag{7}$$

(a)



(b)

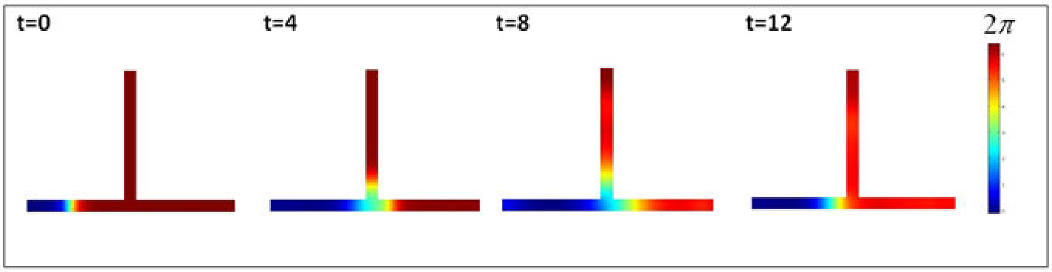


Figure 3: Reflection of mother vortex propagating with velocity $u = 0.99$ when (a) $W = 1 > W_0 = 0.6$ and $u_c = 0.93$. (*b*) $W = W_0 = 0.6$ and $u_c = 0.87$.

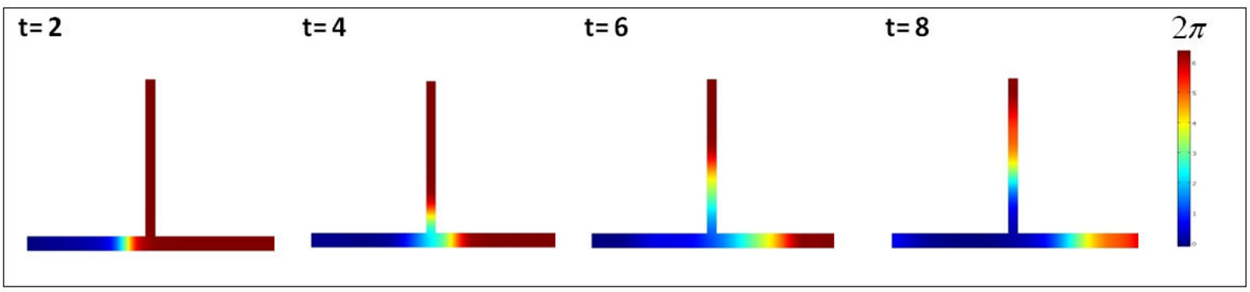


Figure 4: Cloning of a fluxon propagating with velocity $u = 0.85$ when $W = 0.4 < W_0 = 0.6$ and $u_c = 0.8$.

Moreover, we studied the cloning phenomenon in a rotated T-junction by 90^0, which is represented in Fig.1(b) (Farhan and Kusmatsev, 2009a). Our numerical investigations make it obvious that although the two conditions are slightly different from eqs. (4) and (5), the same third condition, eq. (7), should be added to the flux cloning phenomenon as shown in Fig.6 (Farhan and Kusmatsev, 2009b).

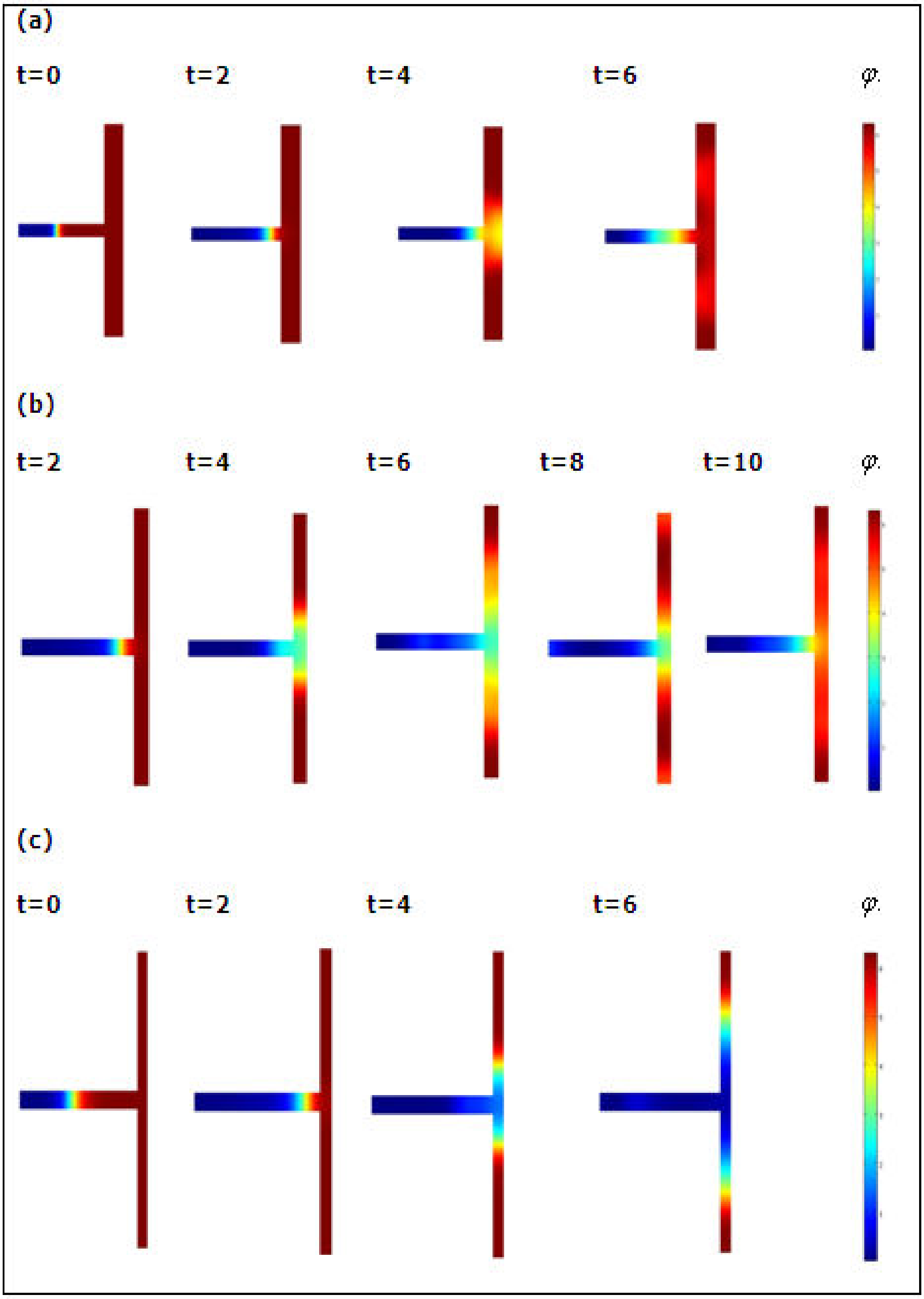


Figure 5: Reflection of moter vortex propagating with velocity $u = 0.99$ when **(a)** $W\!=\!1\!>\!W_0 = 0.6$ and $u_c = 0.95$. **(b)** $W\!=\!W_0 = 0.6$ and $u_c = 0.87$. **(c)** Cloning of a fluxon propagating with velocity $u\!=\!0.7$ when $W\!=\!0.4\!<\!W_0 = 0.6$ and $u_c = 0.66$.

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